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Auxetic spring elements for elastically supporting a sitting or lying

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Abstract: The paper presents a model of an auxetic push spring to be used in mattresses and seats of upholstered furniture. Grid models were subjected to numerical calculations assuming that springs are manufactured from silicone with the Shore hardness of $70^o \pm 5^o$, and design load of 4 kPa. On the basis of calculations it was shown that the structure has a strongly negative Poisson's ratio of -0.41 and that it is perfectly fitted to being combined in parallel systems.

Keywords: push spring, auxetics, numerical calculations

INTRODUCTION

Quality of upholstered furniture to be used as seating and resting furniture is defined by the comfort of their use (Smardzewski and Matwiej 2009, Smardzewski and Wiaderek 2009). It is usually expressed by the level of pressures, contact stresses at the contact of the user's body and the seat or mattress (Smardzewski 2009, Smardzewski and Prekrat 2010). A significant element limiting this comfort is connected with the stiffness of the cushion or mattress. These in turn depend on the rigidity of used foams or springs. Water beds are models for high comfort. They provide uniform body support over a large area. Unfortunately, their use is connected with numerous drawbacks connected with their considerable weight, required heating and exchange of water, significant mobility of their surface and problems with maintenance of cleanliness around this piece of furniture. It is possible to provide uniform body support for the user thanks to the application of an appropriate spring design exhibiting auxetic properties.

RESEARCH OBJECTIVE

The of the study was to develop the design of an auxetic push spring, to determine deflections and stresses in a novel design of springs and to calculate values of Poisson's ratio for the new design.

MATERIAL AND METHODS

Auxetics are materials which are characterised by negative Poisson's ratios (Lakes 1987, Wojciechowski 2003, Wei 2005, Alderson and Alderson 2007). During uniaxial tension the dimensions of the cross-section increase (Fig.1b), while in conventional materials during this test their cross-sections are reduced (Fig.1a).

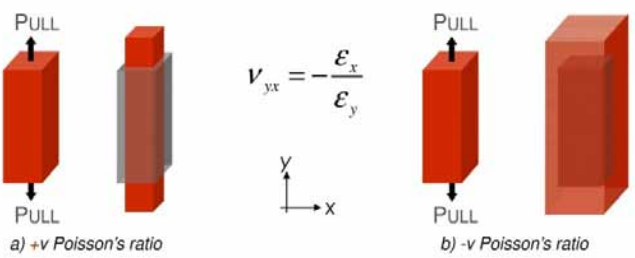


Figure 1. Uniaxial tension test of: a) conventional, and b) auxetic materials.

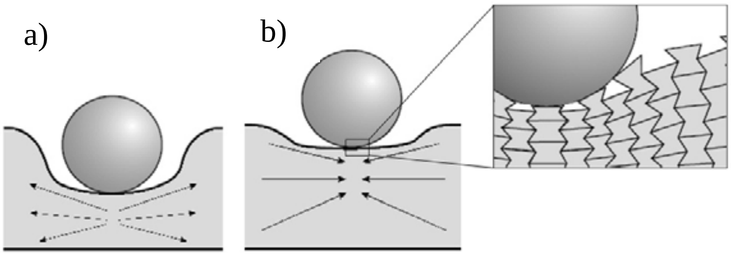


Figure 2. A diagram for the action of a rigid body on: a) conventional, and b) auxetic foundations.

Auxetic characteristics may pertain both to the material and the frame of a structure. In case of contact of a rigid body with a conventional material or frame internal forces cause tensioning of the material (Fig.2a), and thus a disadvantageous distribution of pressures and tangential forces on the contact surface with the body of the user (Fig.3). The parabolic concentrations of reactions of the foundation around ischiatic tubers and the tensioning of the body and clothing by forces of shearing stresses are particularly undesirable. The application of an auxetic material for the foundation (Fig.2b) results in a situation when internal forces in the mattress act inwards and do not strain the body of the user.

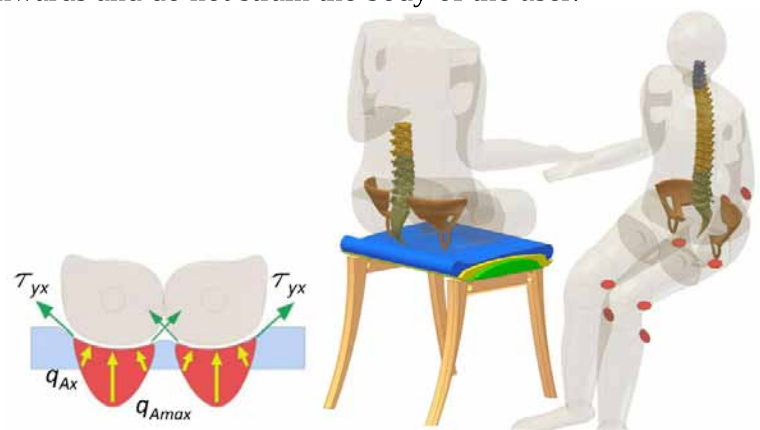


Figure 3. A diagram for the distribution of pressures and friction forces on the body of the user when sitting

The model of a spring as presented in Fig. 4 is a shape exhibiting auxetic properties. It is composed of two rigid sliders of 4 mm in thickness and an elastic foot with wall thickness of 1 mm (Fig.5).

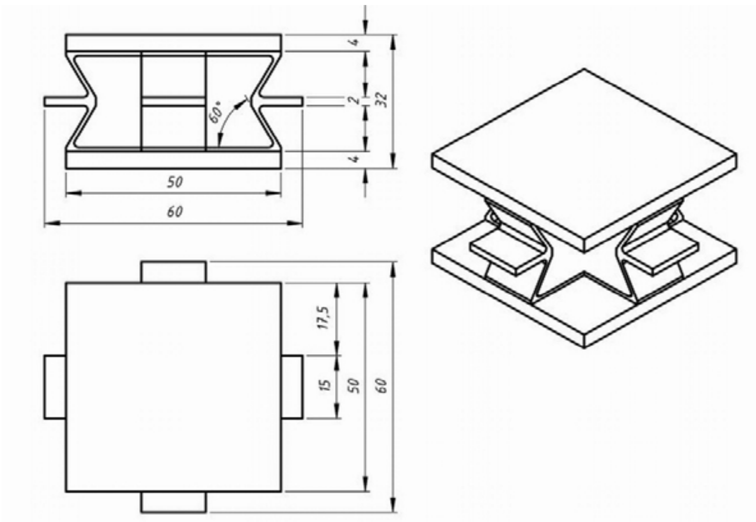


Figure 4. A model of a push spring with auxetic properties

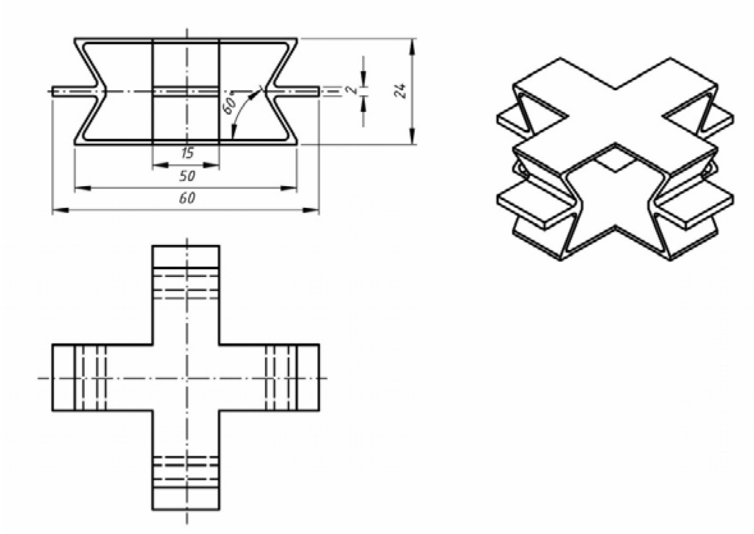


Figure 5. A spring foot

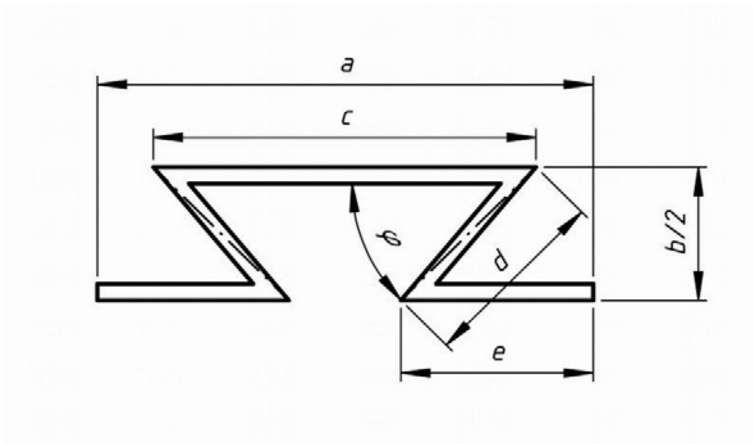


Figure 6. A computational model for the length of foot rockers

Dimensions of sliders resulted from the dimensions of foot rockers and these were determined on the basis of a suboptimization process of the design from Fig. 6, constituting a symmetrical half of the foot from Fig. 5. At the same time it was assumed that width *a* of the spacing of foot rockers of the spring should be $a \in < 60, 90 >$, angle φ between the upper and lower foot rockers should assume the values $\varphi \in < 30^o, 45^o, 60^o, 70^o, 80^o >$, , the length of the lower foot and the length of the skew rocker should be of identical dimensions $e=d$, the length of the upper rocker should be a multiple of the length of the skew rocket while:

$$d = \frac{c}{n}, dla\ n \in < 2{,}3 > .$$

Thus equations were obtained, making it possible to establish the lengths of rockers *b* and *c*, where:

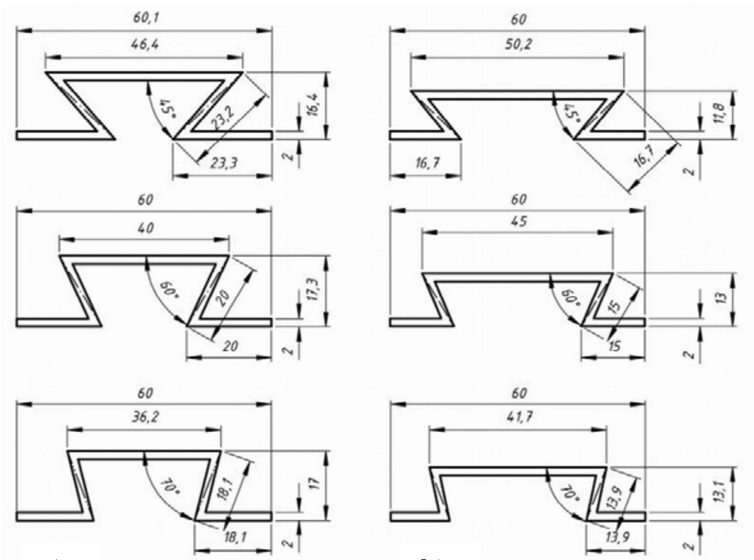
$$b = 2\frac{c}{n}\sin\varphi ,$$

$$c = \frac{an}{n + 2(1 - \cos\varphi)} .$$

Table 1 lists dimensions *b* and *c* established for different values of parameters: *a,n,φ,* while Fig. 7 presents the effect of these parameters on the overall geometry of the spring. It may be observed in this figure that for parameter *n*=2 springs are higher than springs with parameter *n*=3. In turn, the width of the pressure area is greater when *n*=3 and angle *φ* decreases. On this basis the design with the following dimensions and parameters: *a*=60 mm, *n*=3, φ=60^{o}, *c*=45mm, *b*=26mm, was selected for further analyses.

Table 1. A list of dimensions of foot rockers

a (mm)	n	φ(°)	c=f(a,n,φ) (mm)	b=f(a,n,φ) (mm)
60	3	30	55,1	18,4
60	3	45	50,2	23,7
60	3	60	45,0	26,0
60	3	70	41,7	26,1
60	3	80	38,7	25,4
60	2	30	52,9	26,5
60	2	45	46,4	32,8
60	2	60	40,0	34,6
60	2	70	36,2	34,0
60	2	80	32,9	32,4



a) b)

Figure 7. Examples of different foot geometries: a) tall and narrow, b) low and wide.

It was decided to support and load the selected structure as presented in Fig. 8. Due to the proposed use of the modelled spring and its probable application in cushions of seating furniture, the value of load was assumed on the basis of tested contact stresses between the body of the user 50C, at the weight of 75 kg and height of 180 cm, and the seat made of polyurethane foam T3543 of 40 mm in thickness. Tests were conducted using a sensor mat and are presented in Fig. 9 in the form of a stress map. It results from that figure that contact stresses range from 0.28 kPa to 4 kPa. Maximum loads of 4 kPa were assumed for the purpose of numerical calculations.

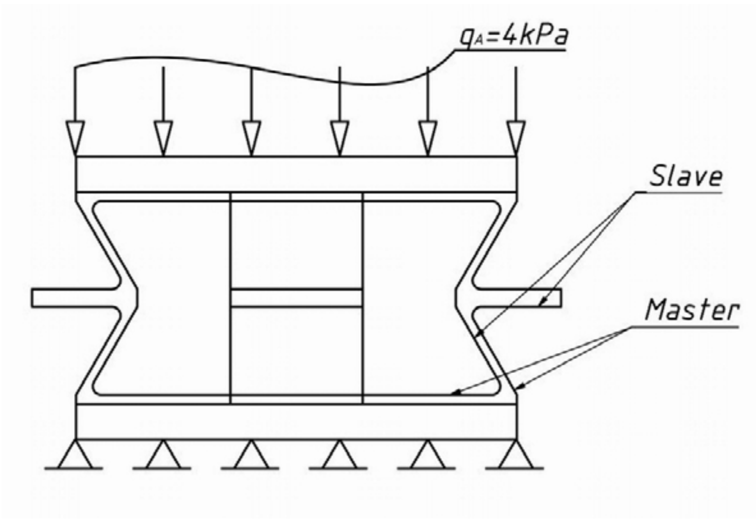


Figure 8. A diagram of loading for a single spring

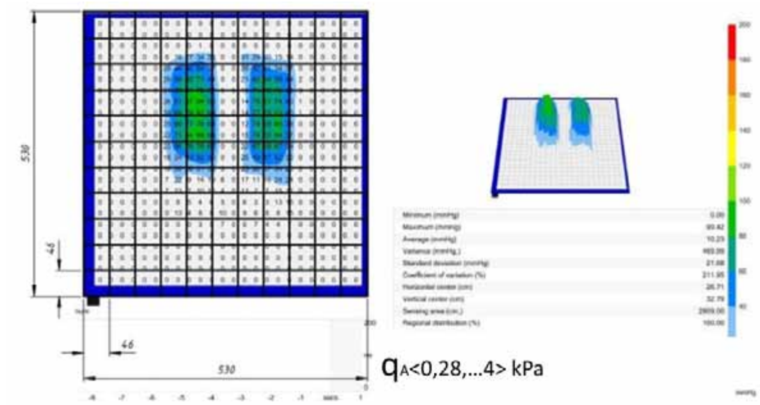


Figure 9. Distribution of pressures on the seat caused by user 50C

It was assumed in the brief foredesign that the modelled spring foot would be made from a hyperelastic material, resistant to temperature changes within the range from -40C^o to +40C^o and of Shore hardness of 70^o±5^o. Silicone was considered most suitable for this purpose, being a non-linear - hyperelastic body (Fig.10).

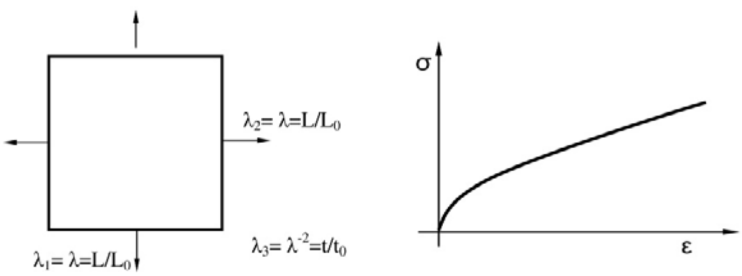


Figure 10. Characteristics of rigidity of silicon as a hyperelastic body

Since silicone is a gum, it was decided for numerical calculations to use the Mooney-Rivlin model, describing energy of elastic deformations of this body in the form:

$$W = C_{10}\left(\bar{I}_1 - 3\right) + C_{01}\left(\bar{I}_2 - 3\right) + \frac{1}{d}\left(J_{el} - 1\right)^2 ,$$

Where:
$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$,
$I_2 = \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2$
$I_3 = \lambda_1^2\lambda_2^2\lambda_3^2$,
$J_{el} = \lambda_1\lambda_2\lambda_3 = \frac{V}{V_0}$
$\mu_0 = 2(C_{10} + C_{01})$,
$\kappa = \frac{2}{d}$.

For the selected silicone with Shore hardness of $70^o \pm 5^o$ the respective coefficients are C_{10}=0.47 and C_{01}=0.12, while the modulus of volume elasticity was k=3.56. It was decided to manufacture the spring slider from hard polyamide with the modulus of rigidity of 12 GPa and Poisson's ratio ν=0.3.

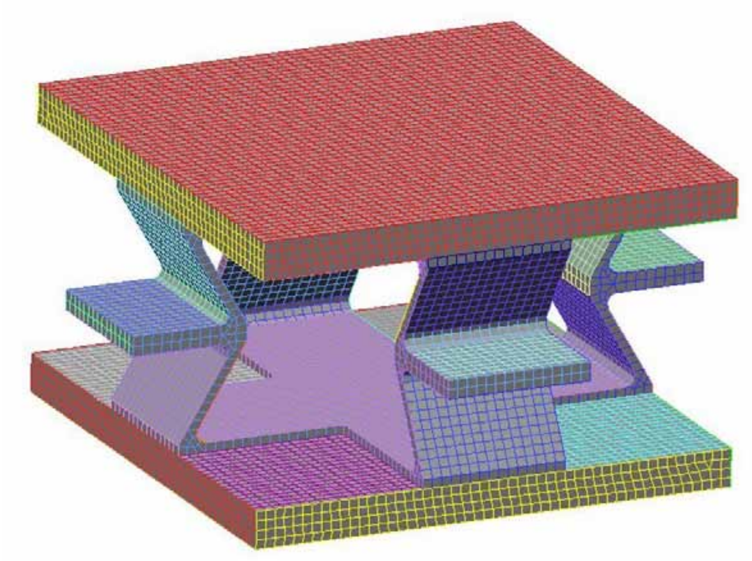


Figure 11. A mesh model of the spring

Numerical calculations were performed in the Algor® system realizing the algorithm of the finite element method. The grid model of the spring is presented in Fig. 11. In its construction 20-node solid finite elements were used. For each pair of surfaces, which may be in contact at compression, contacts were defined by the indication of the master and slave surfaces according to the schema as in Fig. 8. At the same time it was assumed that this contact will be frictionless and the surfaces will have identical rigidity as the material of the spring foot.

RESEARCH RESULTS

Figure 12 presents an image of spring deflection caused by uniaxial compression. As it may be observed from this figure under the load of 2.75 kPa the spring deflects by 15.4 mm and its rockers move backwards inside and press on each other and on the slider. Maximum horizontal displacement towards axes X and Z amounts to 11.2 mm. As it may be observed from Fig. 13 the process of spring settlement is not linear. It results both from the loss of stability of spring rockers as well as contact of individual rocker surfaces at the stage of bifurcation. For a load of approx. 0.8 kPa rockers of the foot frame lose stability and at a slight increment of loads they cause a considerable increment in vertical and horizontal displacements. At the next stage walls of the frame structure press on one another and gradually result in a slight increase in spring rigidity.

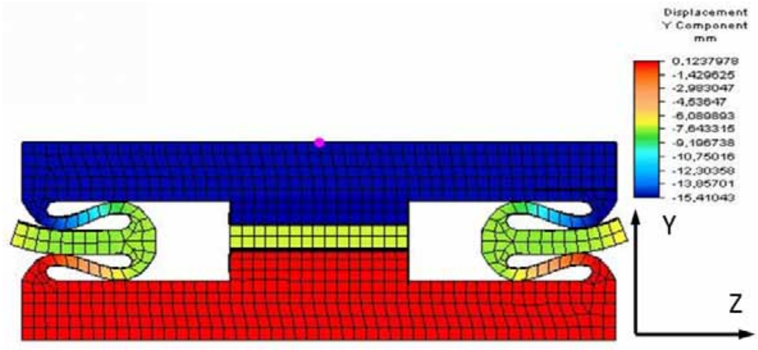


Figure 12. Deflection of a spring in the state of uniaxial compression

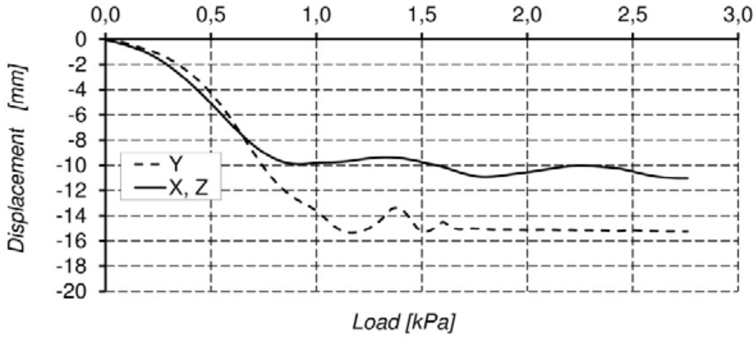


Figure 13. Deflection along the Y axis and displacements along the X, Z axes

On the basis of the shape of the deformed spring it may be concluded that it is an auxetic frame. Figure 14 presents a schema for calculations of Poisson’s ratio for such a structure. The value of this coefficient was calculated from the dependence:

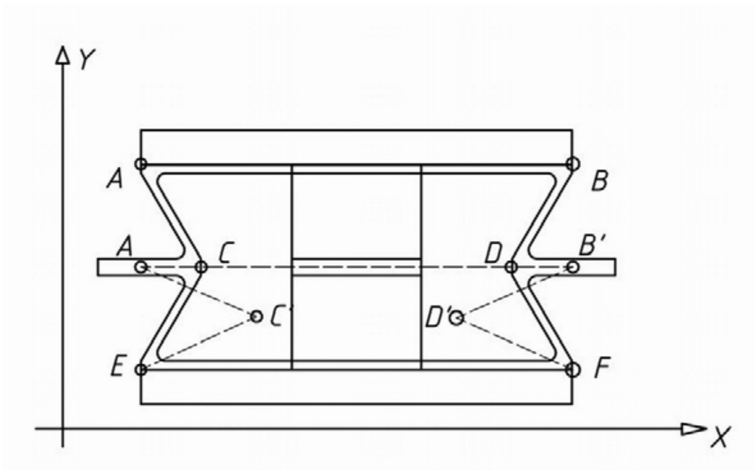


Figure 14. Sketch for calucation of spring Poisson ratio

$$\nu = \frac{\varepsilon_X}{\varepsilon_Y},$$

where:

$$\varepsilon_X = \frac{-\left(\overline{CC'} + \overline{DD'}\right)}{\overline{CD}} = \frac{-(4{,}31 + 4{,}34)}{36{,}28} = -0{,}24,$$

$$\varepsilon_Y = \frac{\overline{AA'}}{\overline{AE}} = \frac{15{,}27}{26} = 0{,}59,$$

$$\nu = \frac{\varepsilon_X}{\varepsilon_Y} = -\frac{0{,}24}{0{,}59} = -0{,}41.$$

As it may be seen, it is a structure with a strongly negative Poisson's ratio, which in a parallel arrangement with other similar structures may provide considerable horizontal displacements. In order to verify the correctness of the design solution it was also decided to estimate the distribution of von Mises reduced stresses caused by design load (Fig.15). The greatest among them appear at sites of rocker contraflexure (point G). However, they are not crippling stresses and they do not exceed 0.7 MPa. As it results from Fig., 16 the course of these stresses changes with the process of loss of rocker stability, described above. Until that moment stresses increase rapidly and almost proportionally. From the moment of the loss of stability and mutual contact of rockers stresses increase slightly.

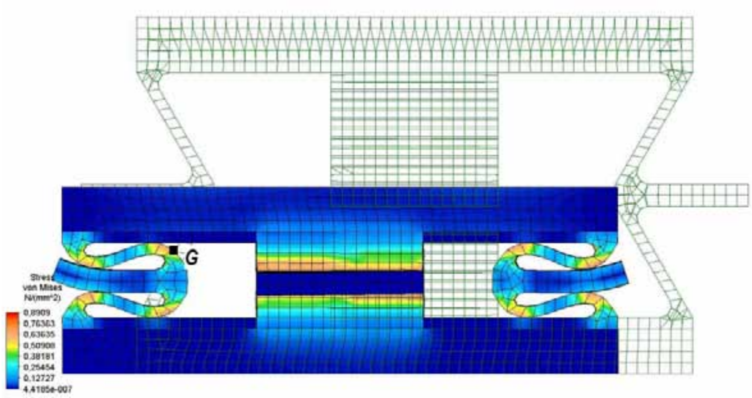


Figure 15. The distribution of von Mises reduced stresses

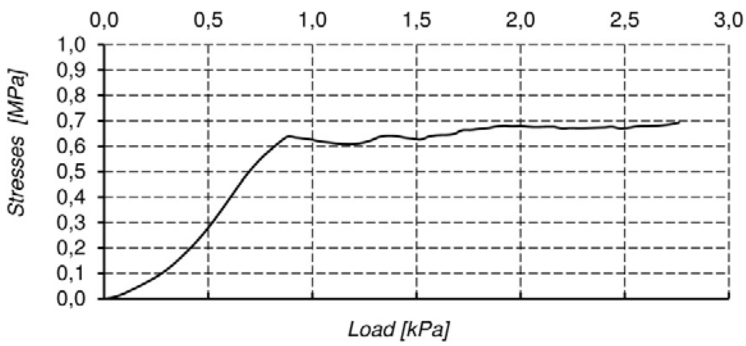


Figure 16. A change in von Mises reduced stresses at point G

CONCLUSIONS

On the basis of the conducted analyses of results of numerical calculations the following conclusions may be formulated:

1. The presented spring model is an auxetic structure with a strongly negative Poisson's ratio,
2. Considerable vertical and horizontal displacements of spring rockers guarantee its applicability in parallel connections,
3. The developed design applied in seat structures should guarantee high comfort of their use.

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Streszczenie: *Auksetyczne elementy sprężyste dla elastycznego podparcia siedzenia i leżenia.* W pracy przedstawiono model naciskowej, auksetycznej sprężyny przeznaczonej do leżysk i siedzisk mebli tapicerowanych. Modele siatkowe poddano obliczeniom numerycznym przyjmując wykonanie sprężyn z silikonu o stopniu twardości $70^o \pm 5^o$ Shor'a, oraz obciążenie użytkowe 4 kPa. Na podstawie obliczeń wykazano, że konstrukcja ma silnie ujemny współczynnik Poissona równy -0,41 oraz , że doskonale nadaje się do łączenia w układy równoległe.

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