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Modeling of joint substitutive rigidity of board elements

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Abstract: *Modeling of joint substitutive rigidity of board elements.* The study presents alternative methods of numerical modeling of dowel joint rigidity of board elements using for this purpose nodes of substitute linear elasticity modulus. Values of joint deflections obtained by way of laboratory experiments and numerical calculations differed by 3 to 4%.

Keywords: dowel joint, substitute linear elasticity modulus, numerical calculations

INTRODUCTION

Virtual furniture design requires verification of their quality from the point of view of their rigidity and strength of the applied construction solutions. This, in turn, is associated with the development of appropriate mesh models of furniture joints in the environment of programs calculating with the assistance of the finite elements method (FEM). Realistic representation of the examined structure in the FEM environment is very labour-consuming, requires numerous corrections of network geometry and meticulous determination of linear elastic properties of the applied materials (Dzięgielewski, Smardzewski, (1996), Kasal (2008), Kasal et al., (2008a,b), Smardzewski (2004a,b), Smardzewski (2005)). However, it is more practical but equally effective to replace joints with semirigid joints (Kłos, Smardzewski (2004), Nicholls, Crisan (2002), Smardzewski, Kłos (2004), Smardzewski, Ożarska (2005), Smardzewski, Prekrat (2002, 2005)). In this paper, an attempt was made to present alternative methods of numerical rigidity modeling of cabinet furniture dowel joints using nodes of substitute linear elasticity modulus.

RESEARCH OBJECTIVE

The aim of this research project was to determine values of the substitute linear elasticity modulus of a spatial dowel joint subjected to closing and opening, to ascertain deflections of this joint, to develop alternative mesh models, to compare the obtained results and to select the model most favourable for the virtual prototyping of furniture.

MATERIAL AND METHODS

Investigations were carried out on angle joints with three 8x32 mm dowel pins of beech wood (Fig.1). Arms of the connections were made of 16 mm thick chipboards and were supported and loaded in accordance with the diagrams presented in Figure 2, measuring values of force P with 0.01 N accuracy and displacements δ_{iP} of point i along the direction of action of force P with 0.01 mm accuracy. For diagrams presented in Figure 2, function equations of the joint arm deflection were determined in the following form:

$$w'' = -\frac{M_y(x)}{EJ}, \text{ where}$$

for $M_y(x) = -\frac{\sqrt{2}}{2}P(l-x)$, it was obtained, $\delta_{iP} = w(x) = \int\left(\int -\frac{\sqrt{2}P(l-x)}{2EJ}dx\right)dx + Cx + D$.

Taking into consideration kinematic boundary conditions and rigidity of joint elements, calculating integration constants and converting the above equation in relation to δ_{iP}, the following equations were obtained:

a) substitute elasticity model in a closing test:

$$E_z^Z = \frac{Pl_z\left(3l_1^2 + 3l_1 l_z + l_z^2\right)}{3\delta_{iP}J_1 - \dfrac{Pl_1^3}{E_1}},$$

b) substitute elasticity model in an opening test:

$$E_z^R = \frac{Pl_z\left(3l_1^2 + 3l_1 l_z + l_z^2\right)}{12\delta_{iP}J_1 - \dfrac{Pl_1^3}{E_1}},$$

where:
l_z – length of the near-node section (for the board, it was assumed as l_z=2h),
l_1 – length of the joint arm,
b – cross-section width of the joint arm,
h – thickness of the joint arm 16 mm,
δ_{iP} – joint total deflection,
J_1 – moment of inertia of joint arm cross-section, $J_1 = \frac{b \cdot h^3}{12}$,
E_1 – Young modulus of the joint arm - chipboard,
P – load ($0.4P_{max} – 0.1P_{max}$).

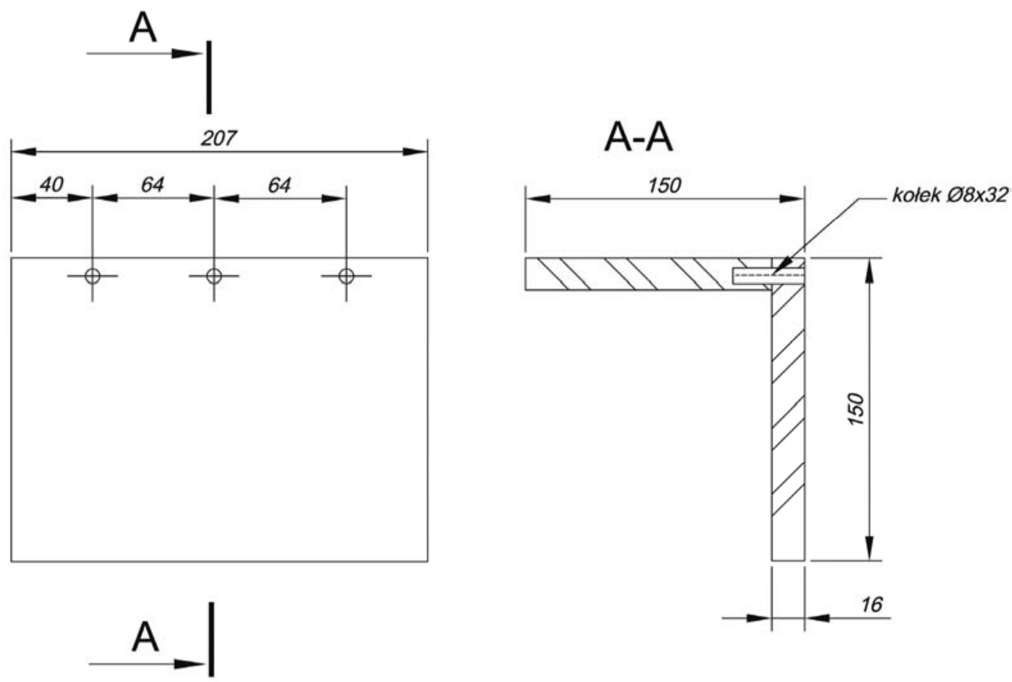
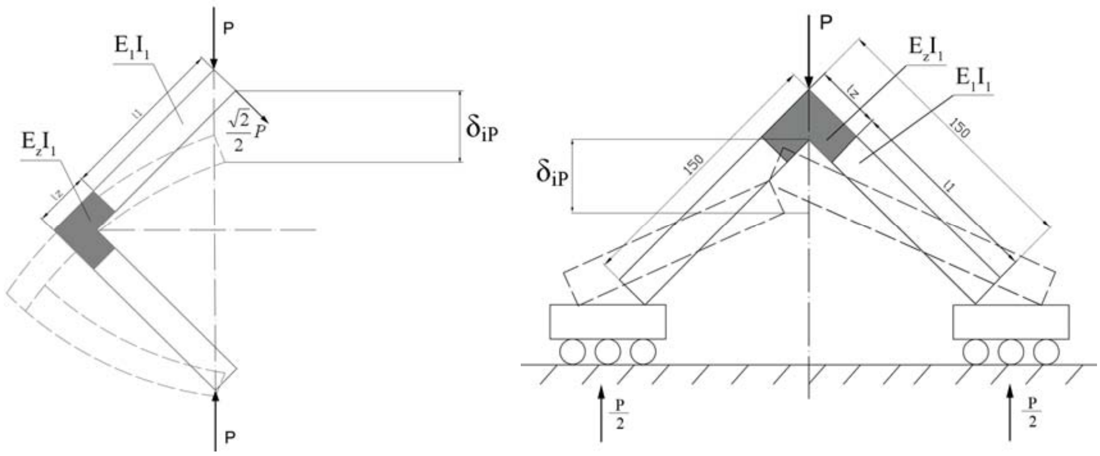


Fig. 1. Dowel joint



a) b)

Fig. 2. Diagram of loading of the angle joint in: a) closing, b) opening tests

Table 1 collates mechanical properties of beech wood, chipboard and glue bond used for the experimental joints, whereas in Table 2, calculation results of substitute modulus of linear elasticity are presented.

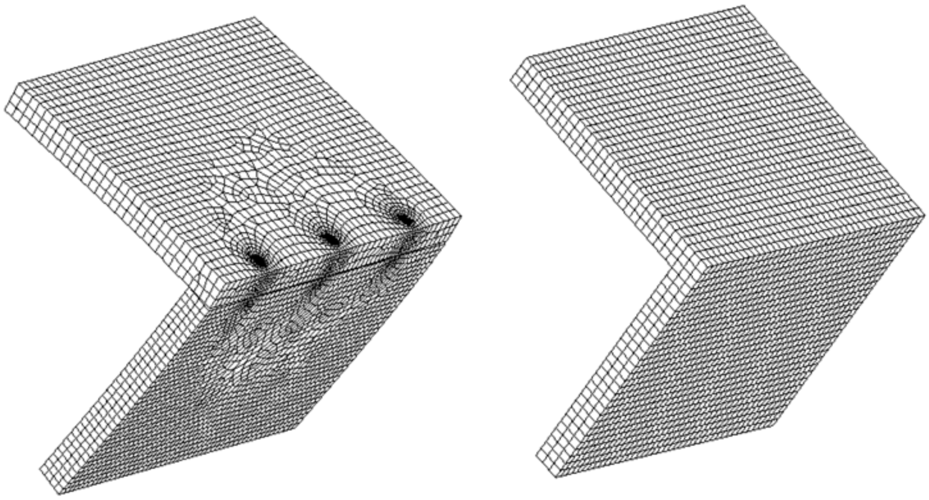
Table 1. Mechanical properties of materials used to make joints

Material	Property	Mean value	Standard deviation	Coefficient of variance [%]
Chipboard	Linear elasticity modulus [MPa]	3660	96.32	2.63
	Bending strength [MPa]	15	0.78	5.18
Beech wood	Linear elasticity modulus [MPa]	18679	1169.75	6.26
	Bending strength [MPa]	124	9.75	7.85
Glue bond	Linear elasticity modulus [MPa]	460	-	-

Table 2. Values of the joint substitute linear elasticity modulus

Kind of research	Mean value [MPa]	Standard deviation [MPa]	Coefficient of variance [%]
Closing test	975	270.08	21.24
Opening test	456	105.89	18.28

When preparing models for numerical calculations, all attempts were made to make sure that geometry and elastic properties attributed to the materials corresponded to real objects as closely as possible. At the same time, issues associated with time necessary for the elaboration of models as well as cost-efficiency of the calculation process were also taken into consideration. For the above reasons, as well as for utilitarian motives, two mesh models were developed for each joint. One of the models constituted a faithful replica of the real joint, while the other, instead of profile and profile-adhesive joints, contained an element of previously determined substitute linear elasticity modulus (Fig.3).



a) b)

Fig. 3. Network of finite elements for a solid body model of angle joint containing: a) dowel pegs, b) element of a substitute modulus of rigidity E_z

Brick type, eight-node elements were used to model the experimental dowel joints (Fig.3a). Joint arms were attributed earlier determined elastic properties of chipboards, while fasteners – elastic properties of beech wood. The glue bond was modeled as an 0.1 mm thick layer of E_s=460 MPa linear elasticity modulus. A contact surface was defined between joint elements differentiating the density of the master and slave networks in such a way that the former was at least two times denser at the place of contact than the slave network. The substitute model of the same joint (Fig.3b) was made using an element of determined substitute linear rigidity. In accordance with the diagram in Figure 2, the substitute element included the near-node sections and arms 2h long (two thicknesses of arms) measuring from the inside part of the connection. The remaining sections of the arms of the joint were attributed elastic properties as in the model from Figure 3a.

Taking into consideration calculated mean values of the linear elasticity substitute modulus of the examined joints (Tab.2) as well as their non-linear rigidity characteristics, the following parameters were used for numerical calculations (Tab.3):

- External load *P* of the values corresponding to 40% of the joint mean breaking load determined for the linear-elastic range,

- Deflection δ_{iP} corresponding to the determined external load,
- Linear elasticity substitute modulus: E^Z_z for the joint closing test and E^R_z for the opening test calculated for the determined *P* and δ_{iP}.

Table 3. Values of loads, deflections and linear elasticity substitute modulus of the experimental joints

Joint	Closing test			Opening test		
	P [N]	δ_{1P} [mm]	E^Z_z [MPa]	P [N]	δ_{2P} [mm]	E^R_z [MPa]
Dowel	141	1.48	975	312	1.56	456

Numerical calculations for closing and opening patterns of the joints were conducted in the environment of Algor® software which realises algorithms of the finite element method. Models from Figure 2 and load values from Table 3 were employed as schemes of static support and loading of the examined systems. The results of calculations comprised illustrations of the distribution of reduced stresses (Mises) in elements of joints, values of these stresses, illustrations of deflections of joints in the direction of action of the external force as well as values of these deflections.

RESEARCH RESULTS

When dowel joints were subjected to closing with force constituting 40% of the breaking load value, it was found that the highest reduced stresses concentrated around fasteners and seats (Fig. 4). The value of these stresses did not exceed 3 MPa which means that the only critical condition deciding about the strength of this construction node was the resistance of the chipboard to delamination (k_r=<1.0MPa).

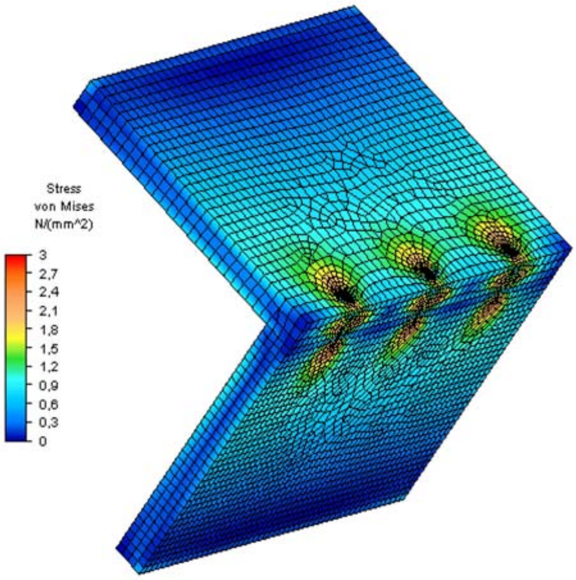
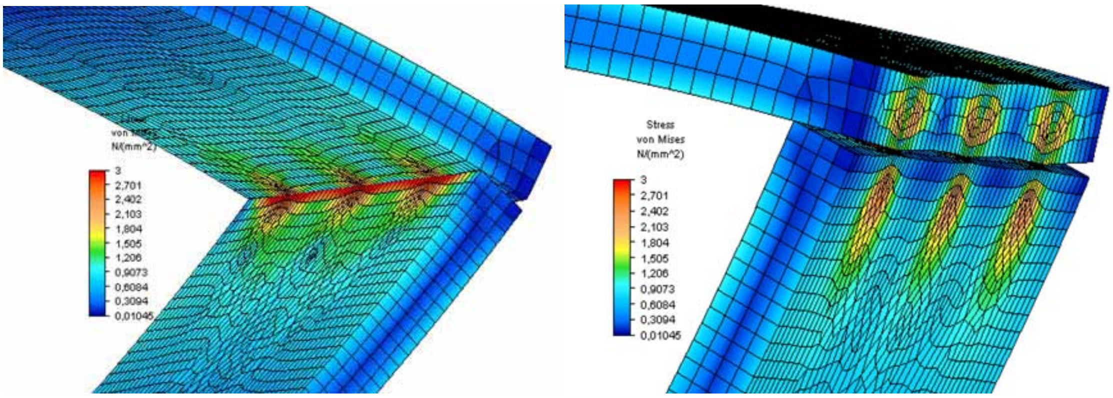


Fig. 4. Distribution of reduced stresses according to Mises in a dowel joint subjected to closing, σ_{max}≈3 MPa



a) b)

Fig. 5. Distribution of reduced stresses according to Mises in a dowel joint subjected to closing at the point of: a) board mutual pressure, b) pressure of fasteners on boards, σ_{max}≈3 MPa

The forms of joint deformations presented in Fig.5 indicate strong loading of the front edge of the joint element and a lack of contact (formation of a gap) between elements on external surfaces (Fig.5b). Because the external chipboard surfaces from which elements of joints were made consist of microchips, contact stresses developed on the edge caused by pressures did not exceed 3 MPa and were not dangerous for this joint.

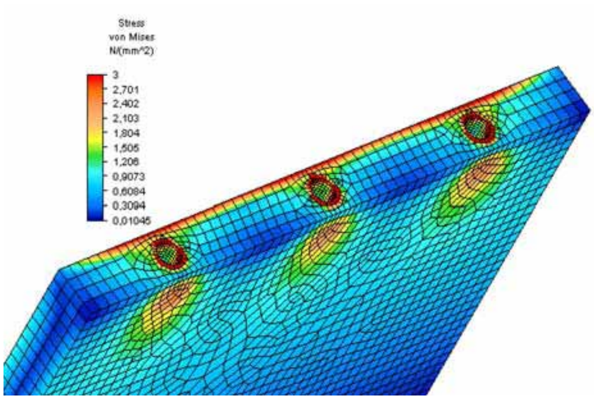


Fig. 6. Distribution of reduced strains according to Mises on the surface of seats in a dowel joint subjected to closing, σ_{max}≈3 MPa

The most dangerous stresses for the load bearing capacity of the joint were those concentrating on the surface of seats (Fig.6). Their value was higher than 3 MPa significantly exceeding the strength of chipboards for stratification. Such concentration of stresses first caused damage of seat surfaces made in the loosest and the weakest zone of the chipboard and then resulted in the delamination of the element in which fasteners were mounted longitudinally to wide planes of the board.

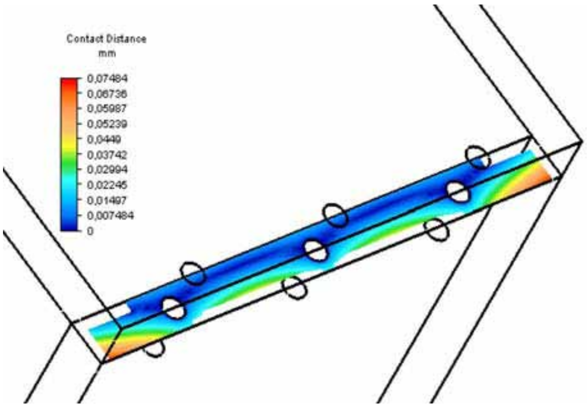


Fig. 7. Size of gap between boards pressing against each other in the joint subjected to closing, δ_{max}≈0,08 mm

It is also interesting that within the discussed range of loads, the joint was characterised by distinct rigidity. The size of the gap formed between board elements of the joint did not exceed 0.08 mm in the course of its closing (Fig.7).

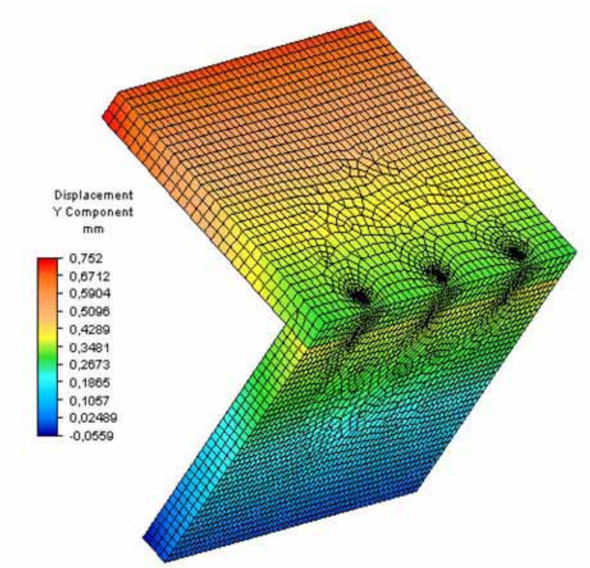
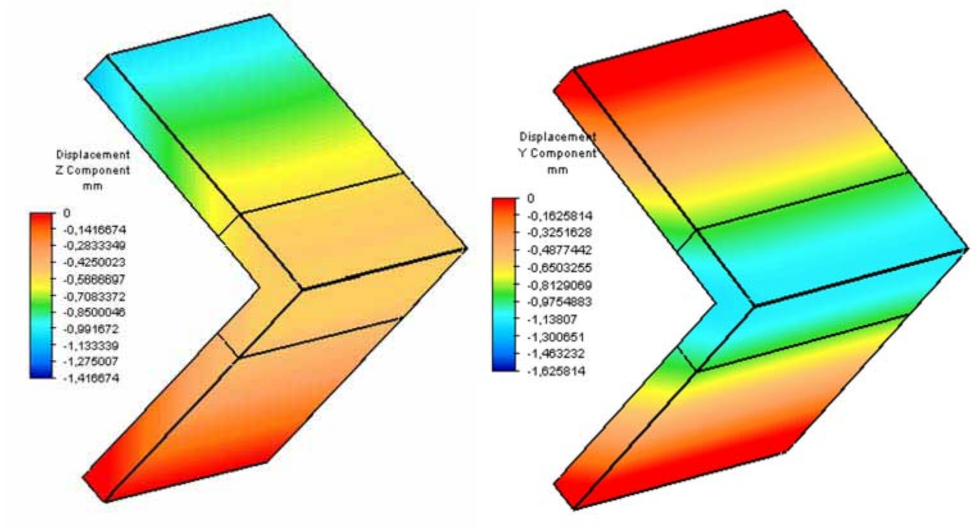


Fig. 8. Deflection value δ_{1P} of the dowel joint subjected to closing, $\delta_{1P(max)}$=0.75 mm.

The quality of the elaborated model was assessed by comparing the calculation results of deflections determined on the direction of load action with the results of laboratory measurements. In the case of numerical calculations, the obtained deflection value amounted to 0.75 mm (Fig. 8). For the adequate substitute model (Fig. 9a), the value of the deflection was 1.42 mm, whereas in the case of the realisation of the pattern causing opening of the joint, the deflection in the direction of force action amounted to 1.62 mm (Fig. 9b). The comparison of the obtained results with measurements obtained experimentally revealed that the results of numerical calculations conducted on models with substitute modulus of linear elasticity were closer to these values.



a) b)

Fig. 9. Size of deflection: a) δ_{1P} of the joint with the element of substitute rigidity ($E^Z{}_Z$) subjected to closing, $\delta_{1P(max)}$=1.42 mm, b) δ_{2P} of the joint with the element of substitute rigidity ($E^R{}_Z$) subjected to opening, $\delta_{2P(max)}$=1.62 mm

Table 4 collates values of deflections obtained from calculations and laboratory experiments after the comparison of the quality of numerical models elaborated for the examined joints. Values of numerical calculations obtained from models faithfully representing the shape of the examined joints were distinctly lower in relation to the results of laboratory measurements. On the other hand, the comparison of the calculation results obtained from models containing nodes with substitute modulus of linear elasticity with empirical results indicates that, in their majority, they were slightly higher. In the case of the wall angle dowel joint, the difference in deflections during closing amounted to 4,1% and during opening – 3,8%.

Table 4. Deflections of the examined joints determined experimentally.

Joint	Closing test δ_{1P} [mm]			Opening test δ_{2P} [mm]		
	Experiment	Brick model		Experiment	Brick model	
		True	Substitute		True	Substitute
Wall	1.48	0.75	1.42	1.56	-	1.62

CONCLUSIONS

Taking into consideration the performed analysis of the obtained research results it should be emphasised that for virtual (numerical) construction modeling of cabinet furniture dowel joints it is recommended to apply cuboid, six- and eight-node networks of finite elements which should be assigned substitute modulus of linear elasticity determined empirically.

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Streszczenie: *Modeling of joint substitutive rigidity of board elements.* W pracy przedstawiono alternatywne sposoby numerycznego modelownia sztywności połączeń kołkowych elementów płytowych z wykorzystaniem węzłów o zastępczym module sprężystości liniowej. Wartości ugięć połączeń, uzyskane w drodze badań laboratoryjnych oraz obliczeń różniły się o 3-4%.

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