# Specific cutting resistance while sawing of wood – the size effect

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**Abstract:** *Specific cutting resistance while sawing of wood – the size effect.* In this paper results of analyses of cutting resistance changes for the cutting models being derived from modern fracture mechanics, which include work of separation in addition to plasticity and friction are presented. Those analyses are conducted for the sawing process on the sash gang saw PRW15M of ash and thermally modified ash. The applied approach allowed us to explain the increase phenomenon of the specific cutting resistance for small depths of cut as a 'size effect'.

*Keywords:* wood sawing, specific cutting resistance, size effect

INTRODUCTION

Energetic effects (cutting forces and cutting power) of wood sawing process are generally calculated on the basis of the specific cutting resistance $k_c$ (cutting force per unit area of cut [1 Fischer, 2 Scholz et al.], which is in the case of wood cutting the function of the following factors: wood species, cutting direction angle (cutting edge position in relation to wood grains), moisture content, wood temperature, tooth geometry, tooth dullness, chip thickness and some others which are less important [3 Agapov, 4 Orlicz]. It is well known that the value of cutting pressure depends heavily on the thickness of the cut (uncut chip thickness) [5 Atkins], which in the case of sawing equivalent is the mean value of feed per tooth $\bar{f}_z$ [6 Ettelt and Gittel, 4 Orlicz, 7 Orlowski and Grzeskiewicz]. The variability of cutting pressure in a function of the mean uncut chip thickness ought to be taken into account in computations for small values of the cut for sash gang saws and bandsaws, and of course in modelling of circular sawing processes, milling or planing, in which variation arises from the process kinematics. Scholz et al. (2009) reported that there are a few mathematical approaches so far known that try to take into account, in more or less simplified way, the effect of specific cutting resistance changes vs. uncut chip thickness. However, neither of those theories explains the causes of that anomalous behaviour.

On the other hand, cutting forces (power) could be considered from a point of view of modern fracture mechanics [5 Atkins]. In this paper, on the basis of "the size effect" [5 Atkins] that is observed also in wood sawing for small depths of cut [7 Orlowski and Grzeskiewicz] is going to be presented.

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**Nomenclature**

$f_z$ – feed per tooth, m
$k_c$ – specific cutting resistance (cutting force per unit area of cut), MPa
$w$ – the width of orthogonal cut equal to $S_t$ (overall set, kerf), m
$F_{cw}$ – cutting force per one tooth during the working stroke, N
$R$ –specific work of surface separation/formation (fracture toughness), Jm$^{-2}$
$St$ – overall set, theoretical kerf, m
$\beta_\mu$ – friction angle given by tan$^{-1}\mu = \beta_\mu$, rad
$\gamma$ – the shear strain along the shear plane
$\gamma_f$ – rake angle, rad

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$\mu$ – friction coefficient
$\tau_\gamma$ – the shear yield stress, Pa
$\Theta_{shear}$ – the friction correction
$\Phi_c$ - shear angle, rad or deg

THEORETICAL BACKGROUND

On the assumption that every saw tooth of the plain shape is symmetrical and sharp, and may have contact with the kerf bottom only during the working stroke of the saw frame, and moreover, the feed per tooth has a uniform distribution in this stroke, the mean cutting force in the working stroke is $\overline{F}_{cw}$ for a single tooth of one saw blade. According to the cutting model [5 Atkins, 8 Orlowski and Atkins] is given by:

$$\overline{F}_{cw} = \left[ \frac{\tau_\gamma S_t \gamma}{Q_{shear}} f_z + \frac{RS_t}{Q_{shear}} \right] \tag{1}$$

where $Q_{shear}$ is the friction correction:

$$Q_{shear} = [1 - (\sin\beta_\mu \sin\Phi_c / \cos(\beta_\mu - \gamma_f)\cos(\Phi_c - \gamma_f))] \tag{2}$$

If the Atkins's model [5, 2009] Eq. (1), which includes fracture toughness $R$, is recast into specific cutting resistance $k_c$ relationship, we obtain:

$$k_c = \frac{\overline{F}_{cw}}{S_t \cdot f_z} = \frac{1}{Q_{shear}} \left( \tau_\gamma \cdot \gamma + \frac{R}{f_z} \right) \tag{3}$$

At small depths of cut the so-called 'size effect' in metal cutting occurs [5 Atkins]. The presence of 'size effect' is also visible in case of sawing, where at small values of $f_z$ decreasing, an increase of $k_c$ occurs [8 Orlowski and Grzeskiewicz]. Thanks to application of the cutting force model, which bases on the modern fracture mechanics, it is now clear that with the $\frac{R}{f_z}$ term on the right hand side the specific cutting resistance must increase at small $f_z$. Furthermore at small values of $f_z$, $\Phi_c$ decreases and $\gamma$ increases [5 Atkins]. Changes in $\Phi_c$ and $\gamma$ vs. $f_z$ will be presented in the next subsection.

THE CASE STUDY

In figure 1 plots of cutting power per one saw and specific cutting resistance $k_c$ in a function of feed per tooth $f_z$ during sawing on the frame sawing machine PRW15M with narrow kerf saw blades (overall set $S_t$ = 2 mm) of ash and thermally modified ash samples are presented [8 Orlowski and Grzeskiewicz].
It has been found out that the thermal modification of ash wood causes a decrease of the specific cutting resistance in comparison to unmodified one. Furthermore for both kind of samples an increase at small $f_z$ has been observed [8 Orlowski and Grzeskiewicz]. Orlowski and Grzeskiewicz [8, 2009] have not explained what was the reason of that phenomenon, however, following the a discussion in the previous subsection, it can be said that what is observed is a pure example of the 'size effect'.
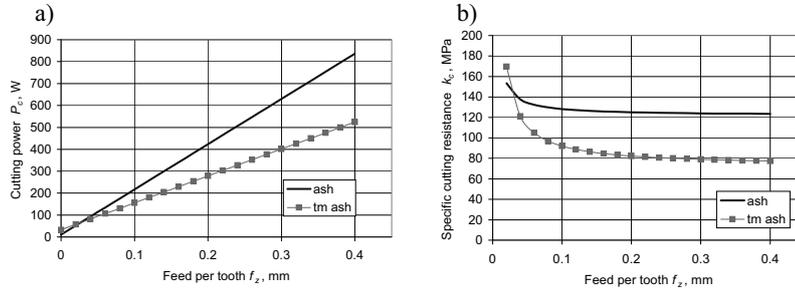
Fig. 1. Cutting power per one saw (a) and specific cutting resistance (b) in a function of feed per tooth during sawing on the frame sawing machine PRW15–M with narrow kerf saw blades (overall set $S_t$ = 2 mm) of ash and thermally modified (tm) ash samples [8 Orlowski and Grzeskiewicz]

On the basis of the experimental data shown in fig. 1a, with the fracture mechanics applied (methodology described by Orlowski and Atkins (2007)), the following material data was calculated for dry ash: fracture toughness is equal to $R$ = 354 J m$^{-2}$ and shear yield strength is equal to $\tau_\gamma$ = 33220 kPa; and for thermally modified ash $R$ = 1090 J m$^{-2}$ and $\tau_\gamma$ = 19350 kPa. These values were used for numerical determination of shear angle $\Phi_c$, with the method proposed by Atkins [9, 2003] (fig. 2a), and the shear strain along the shear plane $\gamma$ (fig. 2b). For lower values of feed per tooth the second term of Eq. (3) plays an increasing role, however, for larger values of feed per tooth its significance rapidly decreases and the main resistance comes from the first term (fig. 3a, b).
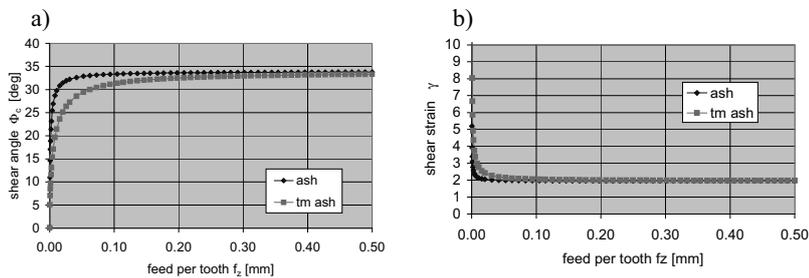


Fig. 2. Comparison of predictions of cutting models that include work of separation in addition to plasticity and friction in the case of sawing dry ash and thermally modified ash on the sash gang saw PRW15M (a) shear plane angle $\Phi_c$ vs. f$_z$, (b) primary shear strain γ vs. f$_z$
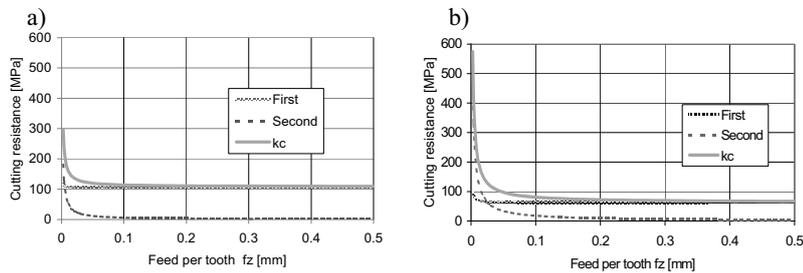


Fig. 3. Cutting resistance in the case of sawing dry ash (a) and thermally modified ash (b) on the sash gang saw PRW15M vs. f$_z$ with its distribution on terms of the Eq. (3), where: First – plot of the first term, Second – plot of the second term, kc – cutting resistance

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CONCLUSIONS

Analyses of changes in cutting resistance predicted by the cutting models derived from modern fracture mechanics, which include work of separation in addition to plasticity and friction, allowed us to explain the reasons for the increase in cutting pressure for small values of feed per tooth. At small depths of cut the so-called 'size effect' in wood sawing occurs which is accompanied with the simultaneous changes of shear plane angle and primary shear strain.

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**Streszczenie:** *Właściwy opór skrawania podczas przecinania drewna – efekt skali.* W artykule przedstawiono wyniki analiz zmian właściwego oporu skrawania, przy zastosowaniu współczesnej mechaniki pękania z uwzględnieniem wiązkości. Analizy wykonano dla procesu przecinania drewna na pilarce ramowej PRW15M drewna jesionu i jesionu modyfikowanego termicznie. Zastosowane podejście pozwala na wyjaśnienie zjawiska wzrostu właściwego powierzchniowego oporu skrawania dla niewielkich grubości warstwy skrawanej jako efektu skali, a nie zmian właściwości materiału obrabianego.

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