

## The prediction method of the shear angle in the cutting zone during wood sawing

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**Abstract:** *The prediction method of the shear angle in the cutting zone during wood sawing.* In this paper the prediction method of the shear plane angle versus uncut chip changes has been presented. The shear plane angle was forecasted with the use of modern fracture mechanics. A nonlinear equation, describing the shear angle as a material dependent value, was solved numerically with the use of iterative Newton-Raphson method. Obtained results of forecasting of the shear plane angle broaden possibilities of energetic effects modelling of the sawing process even for small values of the uncut chip.

*Keywords:* wood sawing, shear angle, fracture mechanics, Newton-Raphson method

### INTRODUCTION

Theoretical and experimental determination of values of forces acting in the cutting process belongs to the basic and simultaneously the most developed field of mechanics of this process. Analyses of cutting forces may be carried out with the classical approach to the cutting energy  $E_c$ , which is composed of deformation and shear energy  $E_{\Phi_c}$ , friction energy at the rake face  $E_\gamma$ , friction energy at the flank face  $E_{\alpha_s}$ , surface energy for the formation of new surfaces  $E_\tau$  and kinetic energy due to chip deflection  $E_M$  [1 Grotte and Antonsson]. As a result of that energetic effects (cutting forces and cutting power) are calculated on the basis of the specific cutting resistance  $k_c$  (cutting force per unit area of cut) [1 Grotte and Antonsson, 2 Orlicz]. On the other hand, cutting forces could be considered from a point of view of modern fracture mechanics [3 Atkins]. The latter was applied for prediction of cutting power during sawing on the sash gang saw [4 Orlowski and Atkins]. The value of the shear angle  $\Phi_c$ , present in the mentioned model which, defines the orientation of the shear plane with respect to cut surface (fig. 1), may be calculated for larger values of feed per tooth  $f_z$  with the classical Merchant's equation (because for large uncut chip values  $\Phi_c = \text{const.}$ , as shown in Atkins's paper (2003)). In this paper, the prediction method of the shear angle for the whole range of uncut chip changes, even for small values, is going to be presented.

### Nomenclature

$f_z$  – feed per tooth, m  
 $w$  – the width of orthogonal cut equal to  $S_i$  (overall set, kerf), m  
 $F_a$  – active force, N  
 $F_c$  – cutting force, N  
 $F_f$  – thrust force (passive), N  
 $F_\mu$  – friction force on the rake face, N  
 $F_N$  – normal force to the rake face, N  
 $F_{T\phi}$  – the force required to shear the wood along the shear plane, N  
 $F_{N\phi}$  – normal force on the shear plane, N  
 $R$  – specific work of surface separation/formation (fracture toughness),  $\text{Jm}^{-2}$   
 $Z$  – the parameter which makes  $\Phi_c$  material dependent  
 $\alpha_f$  – clearance angle, rad  
 $\beta_\mu$  – friction angle given by  $\tan^{-1}\mu = \beta_\mu$ , rad  
 $\gamma$  – the shear strain along the shear plane

$\gamma_f$  – rake angle, rad  
 $\mu$  – friction coefficient  
 $\tau_y$  – the shear yield stress, Pa  
 $\Phi_c$  – shear angle, rad

## THEORETICAL BACKGROUND

Making an assumption that cutting force  $F_c$  acting in the middle of the cutting edge is an equilibrium of forces related to the direction of primary motion for a single saw tooth the mechanical process of material separation from the sawn workpiece, i.e. chip formation, can be described by the example of an orthogonal process (two dimensional deformation) [4 Orłowski and Atkins, 5 Orłowski and Palubicki]. The forces acting on the tooth can be represented in the classical approach by Ernst and Merchant's force circle shown in fig. 1.

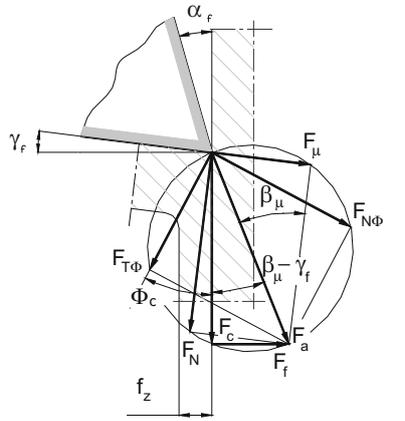


Fig. 1. Simplified cutting process model with Ernst and Merchant's force circle [1 Grotte and Antonsson]

Atkins (2003, 2009) proved that for least force  $F_c$  the shear angle  $\Phi_c$  satisfies:

$$\left[ 1 - \frac{\sin \beta_\mu \sin \Phi_c}{\cos(\beta_\mu - \gamma_f) \cdot \cos(\Phi_c - \gamma_f)} \right] \cdot \left[ \frac{1}{\cos^2(\Phi_c - \gamma_f)} - \frac{1}{\sin^2 \Phi_c} \right] = \quad (1)$$

$$= -[\cot \Phi_c + \tan(\Phi_c - \gamma_f) + Z] \cdot \left[ \frac{\sin \beta_\mu}{\cos(\beta_\mu - \gamma_f)} \left\{ \frac{\cos \Phi_c}{\cos(\Phi_c - \gamma_f)} + \frac{\sin \Phi_c \sin(\Phi_c - \gamma_f)}{\cos^2(\Phi_c - \gamma_f)} \right\} \right]$$

in which:

$$Z = \frac{R}{\tau_y \cdot f_z} \quad (2)$$

is the parameter which makes  $\Phi_c$  material dependent. The solution of nonlinear equation (1)  $f(x) = 0$  ( $x = f_z, f = \Phi_c$ ) cannot be found by the use of analytical functions, therefore was necessary to solve it numerically using the iterative Newton-Raphson method [7 Press et al., 8 Tjalling]. The initial point of computations is very important and decides about convergence of the solution. In this case the value of the shear angle was calculated from Ernst and Merchant equation as an initial point has been assumed:

$$\Phi_c = (\pi/4) - (1/2)(\beta_\mu - \gamma_f) \quad (3)$$

The computational program has been created in the Fortran language.

#### FORECASTING OF THE SHEAR ANGLE

Prediction of the shear angle has been carried out for the case of sawing on the sash gang saw PRW15M [9 Orłowski]. The indispensable data for computation (presented in table 1) was determined according to the methodology described in the work by Orłowski and Atkins (2007). Values of friction coefficients (tab. 1) were taken from the work by Beer (2002).

Table 1. Sawn material data  $R$  and  $\tau_y$ , friction coefficient  $\mu$  and tooth rake angle  $\gamma_f$

Wood species	$R$ $\text{Jm}^{-2}$	$\tau_y$ $\text{kPa}$	$\mu$ –	$\gamma_f$ $\text{deg}$
oak	892	27417	0.8	9
pine	397	22636	0.6	9

Obtained results of predictions of the shear plane angle  $\Phi_c$  vs.  $f_z$  of cutting models that include work of separation in addition to plasticity and friction in the case of sawing dry pine and oak on the sash gang saw PRW15M are presented in fig. 2.

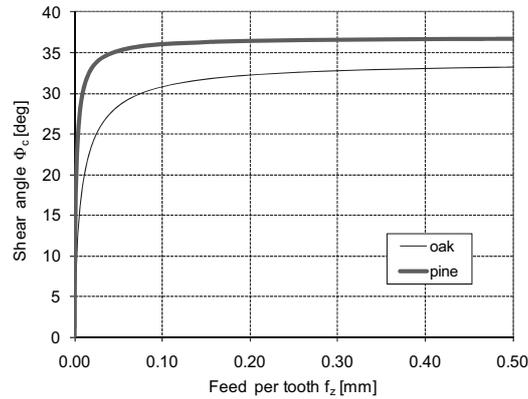


Fig. 2. Predictions of shear plane angle  $\Phi_c$  vs.  $f_z$  in the case of sawing dry pine and oak wood on the sash gang saw PRW15M

#### CONCLUSIONS

Obtained results of forecasting of the shear plane angle for the cutting models being derived from the modern fracture mechanics, which include work of separation in addition to plasticity and friction, broaden possibilities of energetic effects modeling of the sawing process even for small values of the uncut chip.

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**Streszczenie:** *Metoda prognozowania kąta ścinania w strefie skrawania podczas przecinania drewna.* W niniejszym artykule opisano metodę przewidywania kąta ścinania w strefie skrawania dla przypadku przecinania drewna. Kąt ścinania był określany z wykorzystaniem współczesnej mechaniki pękania. Nieliniowe równanie opisujące tą wielkość było rozwiązywane numerycznie za pomocą metody iteracyjnej Newton-Raphson. Otrzymane wyniki obliczeń wykazują możliwość rozszerzenia obszaru modelowania efektów energetycznych procesu przecinania drewna nawet dla niewielkich wartości warstwy skrawanej.

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