An algorithm for determination of threshold value in extruded products by the method of maximum increments: modification of Otsu method

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Abstract: An algorithm for determination of threshold value in extruded products by the method of maximum increments: modification of Otsu method. There are presented the results of comparative analysis of threshold value determination in the extruded products with the use of developed new algorithm. The original and modified Otsu algorithms were compared. In calculations of porosity indices of the extrudate cross section, the threshold values determined automatically with the use of investigated algorithms were compared with indications of an experts’ panel. The results of threshold value calculated with the use of the proposed algorithm were closer to values indicated by the expert panel, than that obtained with the use of comparative algorithms. However, the proposed method tends to overstate the threshold value; in some cases it can cause losing the pores of small lacunarity on the bit map calculated by this method.

Key words: extrudate, porosity, threshold value, comparison of algorithms

INTRODUCTION

Determination of porosity in the extruded products is one of basic investigations carried out to determine sensory and textural features of extrudate. Majority of elaborations connect the extrudate internal structure with defined sensory features, like crispiness or hardness. The porosity can be determined as a number or volume of empty spaces that fill the volume unit [Bourne 1982]. The sample cross section [Ekielski 2011; Ekielski and Żelasiński 2012] is most often described with parameters of its structure (porosity):

- pore density defined as number of empty spaces per unit of cross section area,
- average size of pores per area unit,
- average elongation of empty spaces,
- average orientation of empty spaces,
- complexity of pore structure in extrudate.

Analysis of porosity of sample cross section area is usually performed by determination of pore edges with the use of filtration algorithms [Nadernejad 2008; Singh and Singh 2008]. However, these techniques are not quite useful in fast description of object porosity, mainly due to necessity of closing the edges that describe spaces regarded as pores. Besides, using these methods one cannot determine the space as convex or concave. At present, the methods used most often in porosity determination of surface area are the methods that use the image segmentation into areas regarded as concave. A series of methods involve determination of some threshold value of colour, from which the point is assigned to the selected set that describes features.
of a structure. The basic principle of automatic calculation of threshold value is the automatic selection of optimal level of greyness that allow for isolation of significant objects out of a background, with the use of grayness tint distribution [Sahoo 1988].

Simple analysis of image is usually realized in three stages [Jackman et al. 2009; Ekielski 2011]:
- change of colour image into monochromatic one,
- segmentation of image to two-state image (black-and-white image) by calculation and application of suitable threshold value,
- determination of edges or area of investigated spaces, depending on expectations.

The proper segmentation of image is extremely significant for further calculations. It is one of most difficult problems in processing of digital images. In respect of usability, the automatic or semi-automatic determination of threshold value is applied at present. The automatic determination of threshold value is a very important technique in image segmentation and automatic image processing. The basic principle of automatic threshold value calculation is selection of optimal greyness tint level that allows for isolation of significant objects out of the background, with the use of greyness tint distribution.

The two automatic techniques of threshold value determination can be generally distinguished: determination of general threshold value and determination of local threshold value [Waarsing et al. 2004]. The global threshold value involves selection of a single threshold value from the histogram of the image being processed. The local threshold value uses the local greyness value in selection of many values; each of them is optimal for the small area of image. The global threshold value can be applied easier, but the results are dependent on good (homogeneous) lighting. The local threshold value can be determined at various lighting, but the procedure is complicated and slow.

Among the techniques for determination of global threshold value, the Otsu method is one of more often used [Otsu 1979; Kittler and Illingworth 1985; Kurita et al. 1992]. In this method the threshold value is selected as a point of maximal variability between separated classes of histogram. The Otsu method is optimal in determination the threshold value for large objects out of the background, thus, it is good for determination of threshold value of two-modal or multi-modal distribution. However, it disappoints if the histogram has no local modal value or they are located too close to each other [Fan and Lei 2012; Ekielski 2013]. In the case of extrudate cross section complex structure, the application of this method can lead to improper selection of threshold value. During performing the analysis, the image is represented by two-dimensional (2D) function of greyness intensity – \( f(x,y) \). The value of \( f(x,y) \) is the greyness level that varies from 0 to \( L - 1 \), where \( L \) is the number of different greyness levels. If number of pixels of greyness level – \( i \) amounts to \( n_i \) and \( n \) is total number of pixels of a given image, the probability of hit in the pixel at \( i \)-level of greyness will be equal to:

\[
p_i = \frac{n_i}{n}
\]
Mean value of greyness level for analyzed image was calculated as:

$$\mu_T = \frac{1}{L-1} \sum_{i=0}^{L-1} i \cdot p_i \quad (2)$$

In the case of single threshold value, the pixels are divided into two classes $C_1 = \{0, 1, \ldots, t\}$ and $C_2 = \{t+1, t+2, \ldots, L-1\}$, where $t$ is the calculated threshold value. $C_1$ and $C_2$ correspond to areas of background and image. Probability of assignment of accidental pixel to these classes amounts to:

$$\omega_1(t) = \frac{1}{L-1} \sum_{i=1}^{t} p_i \quad (3)$$

$$\omega_2(t) = \frac{1}{L-1} \sum_{i=t+1}^{L-1} p_i \quad (4)$$

Average value of greyness level for two classes can be calculated with equation:

$$\mu_1 = \frac{1}{\omega_1(t)} \sum_{i=0}^{t} i \cdot p_i \quad (5)$$

and:

$$\mu_2 = \frac{1}{\omega_2(t)} \sum_{i=t+1}^{L-1} i \cdot p_i \quad (6)$$

Using discrimination analysis Otsu proved that optimal threshold value $t^*$ can be calculated by maximization of values between variance of classes that amount to:

$$t^* = \text{ArgMax}_{0 \leq t \leq L} \left\{ \omega_1(t) \cdot (\mu_1(t) - \mu_T)^2 + \omega_2(t) \cdot (2(t) - \mu_T)^2 \right\} \quad (7)$$

The similar but more simple formula was presented by Liao [2001]. The simplified equation that enabled to calculate the threshold value $t^*$ was presented as:

$$t^* = \text{ArgMax}_{0 \leq t \leq L} \left\{ \omega_1(t) \cdot (\mu_1(t) - \mu_T)^2 + \omega_2(t) \cdot (2(t) - \mu_T)^2 \right\} \quad (8)$$

The Otsu method can be easily expanded for multi-level system for determination of the image threshold value. For threshold values $M-1$ that divide image pixels into $M$-classes within the range $[C_1 \ldots C_M]$, the threshold values subjected to optimization are equal to:

$$[t_1^*, t_2^*, \ldots, t_{M-1}^*] = \text{ArgMax} \left\{ \sum_{k=1}^{M} \omega_k \cdot \mu_k^2 \right\} \quad (9)$$

The threshold value calculation method functions well, if histogram of image greyness distribution, for which threshold values are calculated, has no frequent local maxima and minima, thus, there are processed images of histograms characterized by clean bi-modal or multi-modal distributions. The histograms of porous surfaces are often not characterized by bi-modal distribution and even small impurities (e.g. grain residue) can lead to improper setting of threshold value calculated by the Otsu method. Some improvement of threshold value determination can be achieved by application of modification of previously presented
method. This modification involves finding of such greyness value [Fan and Lei 2012], that probability \( p_t \) of obtaining this greyness tint is possibly lowest, but at the same time it is situated between the greyness areas of bigger probability. Therefore, the equation that describes the threshold value calculation algorithm using decreased probability can be written as:

\[
t^* = \text{ArgMax}_{0 \leq t \leq L} \{((1 - p_t) \cdot (\omega_1(t) \cdot \\ \cdot \mu_1^2(t) + \omega_2(t) \cdot \mu_2^2(t)))\}
\]

(11)

In practice, it resolves itself to finding the areas of local probability minimum for appearance of pixels of greyness level \( t \), \( (1 - p_t) \). Graphical interpretation of modified Otsu method for threshold value determination is presented in Figure 1.

In determining the threshold value in porous objects, the modified Otsu method does not often allow for obtaining repeatable equal results. Modifications of this method, proposed in numerous works concerning the image segmentation, cause a substantial complication of calculations [Hou et al. 2006]. These modifications enable, first of all, to carry out the area plucking out process so, to obtain the edge image existing in investigated image. They are used in the face identification systems, registration plates and other objects, where recognizing of lines is significant. One can find on the

FIGURE 1. Analyzed: a – one-modal image, b – multi-modal image, and determined threshold values for: c – one-modal image, d – multi-modal image distributions by Otsu method (1) and modified Otsu version – valley emphasis (2)
market several commercial programs that enable – through multi-threshold segmentation – to get information on the investigated areas depth, based on the modified multi-threshold method for threshold value determination. Therefore, there exists a method for the third dimension approximation by analysis of two-dimensional analysis.

RESULTS AND DISCUSSION

In this work there was proposed the own relatively quick method for threshold value calculation during analysis of porous materials. This method is generally similar to the modified Otsu method, however, it is more useful in determination of dense areas in the porous structures. If image is a set of \( M \)-points of \( N \)-greyness tints, every point of image can be classified into one of \( N \)-classes of greyness level \( t_n \) and size \( n_n \). Therefore, introducing the threshold value at level \( t_k \), one can divide the set \( M \) into two sets:

\[
C_{ik} = \sum_{i=0}^{k} n_i
\]

where:
- \( n_i \) – size of class determined by greyness tint at level \( i \) \( (i < 0.255) \);
- \( C_k \) – size of pixel set separated with the use of threshold value \( k \).

Practically, it means classification of pixels as white. Thus, number of pixels classified as black (in two-state system) can be calculated with equation:

\[
C_{\text{cl}k} = M - C_{ik}
\]

Therefore, if there is introduced the function describing relative size of white pixels depending on applied threshold value:

\[
L_k(t) = \frac{C_{ik}}{M} = \frac{\sum_{i=0}^{k} n_i}{M}
\]  

(14)

It is possible to investigate distribution of greyness levels in the investigated image. Therefore, if function \( L_k(k) \) is linear, it means existing of multi-modal distribution of greyness levels of low diversification of values. In practice, it means low diversification the image and background elements. More information is carried by the value that describes a change in the number of white pixels during transition between subsequent cutting off levels:

\[
\Delta L_k(t) = \frac{\sum_{i=0}^{t} n_i - \sum_{i=0}^{t+1} n_i}{M}
\]  

(15)

Values of function \( \Delta L_k(t) \) represent an increment in number of pixels counted as white during transition between subsequent cutting off levels. The subsequent local minima point out at “losing” of information on subsequent homogeneous areas. If the values of changes in subsequent cutting off levels are normalized, the local maxima can be used in determination of porous structure depth (darkening degree). It is useful in making the quasi-three-dimensional description of two-dimensional images. Generalization of the equation consists in its connecting with an increment in cutting off values and determination of threshold value in this form. In order to determine threshold values, the function \( P(k) \) was introduced that determined the local changes in value of function \( \Delta L_k(t) \). These values are described as:
The threshold value calculated as:

\[ t = \text{Max}_{0 < t < k} (P_k) \]  

(17)

describes the area of maximal change in relative increment (positive) of the set of samples classified into one of variability areas during segmentation. This method is similar to valley algorithm and the difference (when compared to the modified Otsu method) consists in analyzing of relative change in the set size, instead of the change in cumulative distribution function of probability.

**MATERIAL AND METHODS**

Verification of the method was carried out by analysis of images obtained in investigations on internal structure of the wheat-maize extrudate. The cross sections of samples were photographed under the stereoscopic microscope equipped with CCD camera of resolution 6 MPx.

Then, according to previously presented procedure, the coloured cross section image was converted to monochromatic space. The conversion was executed due to the algorithm written in Matlab program. The monochromatic image matrix was presented in the form of histogram, on which the size of pixels of greyness degrees was placed. Since the sector of image was analyzed, no extreme greyness tint values occurred. Figure 3 presents the analyzed image and the histogram of distribution of greyness parameter value for pixels included in the image.

The histogram presented in Figure 1b represents the multi-modal distribution of pixels size. In Figure 1 there is presented the course of changes in functions \( L_{g}(k) \) and \( P(k) \), depending on \( k \)-value that determine \( s \) – subsequent values of greyness level. The first local maximum was obtained for \( k_1 = 75 \). Both functions \( L_{g}(k) \) and \( P(k) \) reach the local maxima in point \( k_1 \).

The threshold value was calculated with the use of Otsu algorithm, the modified Otsu algorithm and own developed algorithm of the highest increments. The threshold values determined by mentioned methods are presented in Figure 2.
The cutting off value (threshold value) calculated by modified Otsu algorithm amounted to $k_a = 88$, by the highest increment method $k_b = 105$ and by classic Otsu method $k_c = 75$. The result of classic Otsu method is the lowest, due to determination of threshold value in multi-modal system and unsuitability of algorithm to calculations of multi-modal systems.

The threshold value determined automatically were verified by the set of experts consisted of nine persons, who determined the threshold value independently of each other. The threshold value was changed manually during evaluation. Every expert had an independent system and was able to change the threshold value in the range from 0 to 255 with unitary increment. Not knowing the results of calculations performed by automatic system for threshold value determination, the experts determined the optimal value (according to their assessment), at which the two-state image best represented the monochromatic image structure. The effect of manual setting of threshold value on two-state diagram (bit map) is presented in Figure 5.

The obtained results of threshold value determination in manual segmentation were include in interval $t \in <85; 107>$. The median of indications amounted to $t = 102$.

The determined threshold value were used in execution of two-state segmentation of the obtained image of extruded sample cross section (Fig. 6).
CONCLUSIONS

The developed algorithm for threshold value calculations used in investigations on sample cross section porosity was closest to that indicated by the set of experts. The values calculated by modified Otsu method were also included in value range indicated by the set of experts, however, the value calculated by proposed algorithm is closer to median value of expert set indications. It means better fitting of algorithm to expectations and perception of human sight. However, the presented method, due to a simple algorithm, tends to overstate the threshold value; it can cause losing the pores values of small lacunarity. It does not depreciate its usability in quick scanning of porous surfaces.

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